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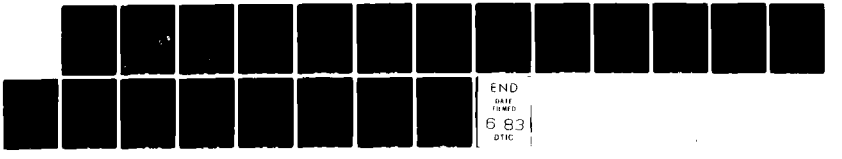

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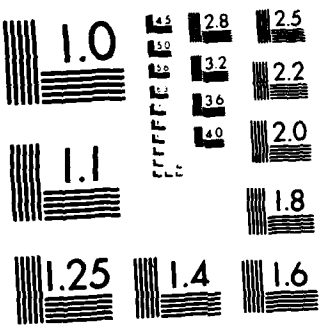

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The turbulent equations for a neutral fluid are derived from the standpoint of Statistical Mechanics. The Three-Point Method of turbulence was investigated for limitations on the theory. Turbulent equations applicable to the adiabatic response of a plasma are developed.

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FINAL REPORT

Contract No. N00014-75-C-0473

STATISTICAL MECHANICS OF NON-EQUILIBRIUM PROCESSES IN PLASMAS AND GASES

PHYSICS BRANCH

THE OFFICE OF NAVAL RESEARCH

MAY 1983

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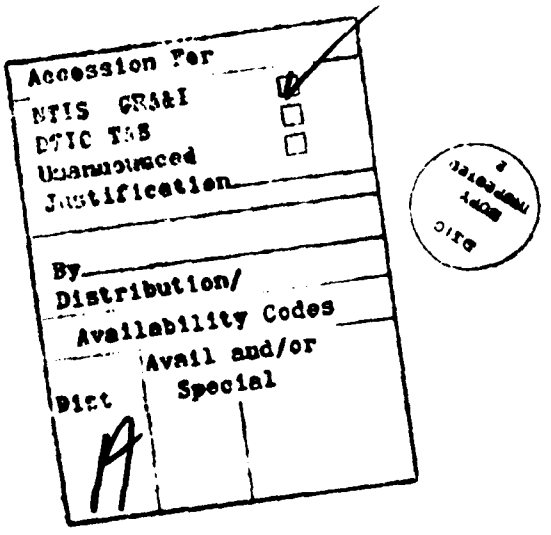
TABLE OF CONTENTS



I) GENERAL

This is a final report of contract number N00014-76-C-0089 on the Statistical Mechanics of Non-Equilibrium Processes in Plasmas and Gases.

II) DESCRIPTION OF RESEARCH

Since fluids, both plasma and neutral, are almost always turbulent, the turbulent fluid plays an important role in a wide range of phenomena.

The work in neutral fluids had been mainly from the standpoint of the fluid equations (Navier-Stokes equation). We have approached the problem of obtaining the basic equations from the standpoint of statistical mechanics[1,2] (Liouville equation) as the solid foundation of fluid mechanics.[3] The central problem common to all approaches is the closure of an infinite set of coupled equations. A recent approach[4] using the fluid equations employes a three-point Green's function $G^{(3)}(t,t')$. The basic assumption in this work is that the ensemble average $\langle G^{(3)}(t,t')\rangle$ approaches zero as $(t-t') > \tau_c$ where τ_c is a time on the order of the decay time of the total energy. This property is used to obtain closure. A result of this theory is that the evolution of the energy is essentially independent of the ternary correlation function $\langle T\rangle \equiv \langle \underset{\sim}{v}(\underset{\sim}{r}_1,t)\underset{\sim}{v}(\underset{\sim}{r}_2,t)\underset{\sim}{v}(\underset{\sim}{r}_3,t)\rangle$. We show[5] (Appendix A) that this conclusion is not valid in general. The problem most likely rests on the fact that $\langle G^{(3)}(t\ t')\rangle$ is not independent of the initial value of $\langle T\rangle$ as was implicitly assumed in[4].

Recent work on the Adiabatic Modifications to Plasma Turbulence Theories[6] introduces a special perturbation theory to account for the effects of the adiabatic response of a plasma. We have shown[7]) that the conventional turbulent perturbation theory[8], if properly used, produces the modified results (Appendix B).

III) PERSONNEL

The chief investigator on this project was Professor Marvin B. Lewis of the Department of Mechanical and Nuclear Engineering and the Department of Physics and Astronomy of Northwestern University.

IV) CONTRACT PUBLICATIONS

"Kinetic Theory of Turbulent Flows," Ref. (1)

"Kinetic Theory of Turbulent Flows II," Ref. (2)

"Three-Point Method in Turbulence." Ref. (5) (submitted for publication - Physics of Fluids)

Comment on "Adiabatic Modifications to Plasma Turbulence Theories," Ref. (7) (to be published June 1983, Physics of Fluids)

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APPENDIX A

Three-Point Method in Turbulence

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The three-point method for the description of turbulent flows rests on the assumption that the average of the three-point Green's function approaches zero. The argument is made that the evolution of the energy is essentially independent of the initial value of the ternary correlation function. It is shown that there exist initial conditions for which this conclusion does not hold for times up to the viscous decay time.

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I. Introduction

Recent work on the theory of turbulence[1] employes the use of a three-point Green's function $G^{(3)}(t,t')$. The assumption is made that the ensemble average $\langle G^{(3)}(t,t')\rangle$ approaches zero for $(t-t')>\tau_c$. This property, due to the nonlinear effects, introduces further damping in addition to the viscous damping into the system. On the basis of this assumption a closed equation for the energy $\langle U\rangle$ is obtained. This equation explicitly contains the initial value of the ternary correlation function $\langle T\rangle$ which is said to be arbitrary. The argument is made that, since the dependence on the initial $\langle T\rangle$ goes to zero due to the assumed property of $\langle G^{(3)}\rangle$, the initial $\langle T\rangle$ can be neglected even if it is not zero. This conclusion is that the evolution of $\langle U\rangle$ is essentially independent of the initial $\langle T\rangle$.

We show that there exist ensembles for which the conclusion does not hold for a time t_o from the initial time where $2t_o$ is any time less than the viscous decay time.

II. Summary of the Three-Point Method[1]

The Navier-Stokes equation for an incompressible fluid can be written as

$$[\frac{\partial}{\partial t} + L_o(\underset{\sim}{r})+L(\underset{\sim}{r},t)]\underset{\sim}{v}(\underset{\sim}{r},t) = 0 \qquad (1a)$$

where

$$L_o \equiv -\mu\nabla^2 \quad , \qquad (1b)$$

$$L(\underset{\sim}{r},t)\underset{\sim}{v}(\underset{\sim}{r},t) \equiv \int d\underset{\sim}{r}_a A(\underset{\sim}{r},\underset{\sim}{r}_a): \underset{\sim}{v}(\underset{\sim}{r}_a,t)\underset{\sim}{v}(\underset{\sim}{r}_a,t) \quad . \qquad (1c)$$

The equation for the energy (covariance correlation function) $\langle U(\underset{\sim}{r}_1,\underset{\sim}{r}_2,t)\rangle \equiv \langle\underset{\sim}{v}(\underset{\sim}{r}_1,t)\underset{\sim}{v}(\underset{\sim}{r}_2,t)\rangle$ for homogeneous systems with $\langle\underset{\sim}{v}(\underset{\sim}{r},t)\rangle = 0$ is

$$[\frac{\partial}{\partial t} +L_o(\underset{\sim}{r})+L_o(\underset{\sim}{r}_o)]\langle U(\underset{\sim}{r},\underset{\sim}{r}_o,t)\rangle = -(1+tr)\int d\underset{\sim}{r}_a A(\underset{\sim}{r}_o,\underset{\sim}{r}_a): \langle T(\underset{\sim}{r}_a,\underset{\sim}{r}_a,\underset{\sim}{r},t)\rangle \quad , \qquad (2)$$

where tr is the transpose and $T(\underset{\sim}{r}_1,\underset{\sim}{r}_2,\underset{\sim}{r}_3,t) \equiv \underset{\sim}{v}(\underset{\sim}{r}_1,t)\underset{\sim}{v}(\underset{\sim}{r}_2,t)\underset{\sim}{v}(\underset{\sim}{r}_3,t)$, $\langle\ \rangle$ is ensemble average and $\langle T\rangle$ is the ternary correlation function. The basic problem is to obtain an approximate expression for $\langle T\rangle$ in terms of $\langle U\rangle$ so that Eq. (2) is a closed equation.

The development of the three-point method introduces the Green's function G defined by

$$\underset{\sim}{v}(\underset{\sim}{r},t) = \int d\underset{\sim}{r}' G(\underset{\sim}{r},t;\underset{\sim}{r}'t')\cdot\underset{\sim}{v}(\underset{\sim}{r}',t') \quad . \tag{3a}$$

Another approach is to use the propagator[2] W defined by

$$\underset{\sim}{v}(\underset{\sim}{r},t) = W(\underset{\sim}{r},t,t')\underset{\sim}{v}(\underset{\sim}{r},t') \quad , \tag{3b}$$

where $W(\underset{\sim}{r},t',t') = 1$. The equation for W is the same as the equation for $\underset{\sim}{v}$;

$$(\frac{\partial}{\partial t} + L_o + L)W = 0 \quad . \tag{4}$$

The evolution of $T(\underset{\sim}{r}_1,\underset{\sim}{r}_2,\underset{\sim}{r}_3,t) \equiv T(x,t)$ is given by

$$T(x,t) = W^{(3)}(x,t,t')\cdot T(x,t') \quad , \tag{5}$$

Where $W^{(3)}$, the three-point propagator, is defined by

$$W^{(3)}(x,t,t') \equiv W(\underset{\sim}{r}_1,t,t')\ W(\underset{\sim}{r}_2,t,t')\ W(\underset{\sim}{r}_3,t,t') \quad , \tag{6}$$

where it is understood that $W(\underset{\sim}{r}_1,t,t')$ acts only on functions of $\underset{\sim}{r}_1$. The equation for $W^{(3)}$ follows from (4), i.e.,

$$[\frac{\partial}{\partial t} + L_o^3(x) + L^3(x,t)]W^{(3)}(x,t,t') = 0 \quad , \tag{7a}$$

where

$$L_o^3(x) = L_o(\underset{\sim}{r}_1) + L_o(\underset{\sim}{r}_2) + L_o(\underset{\sim}{r}_3) \quad , \tag{7b}$$

$$L^3(x,t) = L(\underset{\sim}{r}_1,t) + L(\underset{\sim}{r}_2,t) + L(\underset{\sim}{r}_3,t) \quad . \tag{7c}$$

What is needed in (2) is $\langle T\rangle$, i.e.,

$$\langle T(x,t)\rangle = \langle W^{(3)}(x,t,t')T(x,t')\rangle \quad . \tag{8}$$

The method proceeds with an expansion of $G^{(3)}$ in terms of $\langle G^{(3)}\rangle$ [Ref. 1, Eq. 23]. Here we work with $W^{(3)}$. The corresponding expansion for $W^{(3)}$ is

$$W^{(3)}(t,t') \equiv \langle W^{(3)}(t,t')\rangle + \delta W^{(3)}(t,t') \quad , \tag{9}$$

We then have from (5) and (9)

$$T(t) = \langle W^{(3)}(t,t')\rangle T(t') + \delta W^{(3)}(t,t')T(t') \quad . \tag{10}$$

The ensemble average of T is

$$\langle T(t)\rangle = T_{Iv}(t) + T_{hc}(t) \quad , \tag{11a}$$

$$T_{Iv}(t) \equiv \langle W^{(3)}(t,t')\rangle\langle T(t')\rangle \quad , \tag{11b}$$

$$T_{hc}(t) = \langle \delta W^{(3)}(t,t')T(t')\rangle \quad . \tag{11c}$$

The term T_{hc} is broken up into two terms

$$T_{hc}(t) \equiv T_c(t) + T_h(t) \quad , \tag{12a}$$

where T_c is given by

$$T_c(t) = -\int_{t'}^{t} dt''\langle W^{(3)}(t,t'')\rangle\langle L^3(t'')T(t'')\rangle \quad , \tag{12b}$$

III. The Closed Energy Equation[1)]

The expression for $\langle T\rangle$ has three parts

$$\langle T(t)\rangle = T_{Iv}(t) + T_c(t) + T_h(t) \quad , \tag{13}$$

where T_{Iv} is called the initial value term because of its dependence on the initial value of $\langle T\rangle$, T_c is the cascade term and T_h is the remainder term. The right-hand side of (2) has corresponding parts S_{Iv}, S_c, and S_h.

The assumption employed is that $\langle W^{(3)}(t_2,t_1)\rangle$ approaches zero as (t_2-t_1) increases with a characteristic time τ_c that is small compared with the viscous decay time. The consequences of this assumption are:

a) The T_h term can be neglected.

b) That $\langle v\ v\ v\ v\rangle$ that occurs in S_c can be written as a sum of terms of the form $\langle U\rangle\langle U\rangle$. This result reduces Eq. (2), except for the term $\langle W^{(3)}\rangle$ in T_c [Eq. 12b], to an equation for $\langle U\rangle$. The closure is completed with an approximate equation for $\langle W^{(3)}\rangle$ that is closed [Ref.

1, Eq. 70]. Although no real estimate was made for τ_c it would appear that if τ_c is greater than the decay time for the total energy then, as in the usual quasi-normal theory[3], the energy spectrum can become negative[4].

c) The initial value term T_{Iv} is negligible after a time interval $(t-t')>\tau_c$. Since eq. (2) for $\langle U\rangle$ contains the initial value of $\langle T\rangle$ in the term S_{Iv}, it can be arbitrary.

On the basis of (c) the argument is then made that T_{Iv} can be neglected even if the initial value of $\langle T\rangle$ is not zero since it decays rapidly to zero. This conclusion is that the evolution of $\langle U\rangle$ is essentially independent of the initial $\langle T\rangle$. We shall show that this conclusion is not true in general for a time t_o from the initial time where $2t_o$ is any time less than the viscous decay time. We compare two systems that are related by velocity reversal. These two systems initially have the same $\langle U\rangle$ but one $\langle T\rangle$ is the negative of the other.

IV. Reversibility of the Inviscid Equations

We consider the evolution of a system from the initial time $t'= 0$ to a time $(2t_o)$. For large Reynolds number and for $\langle U\rangle$ restricted to macroscopic scales at the initial time, the viscous decay time is larger than the decay time for the total energy and over the interval $(0, 2t_o)$ the effects of viscosity can be neglected. Over this time interval the Navier-Stokes equation is reversible. Consider the system to have a velocity $\underset{\sim}{v}_+ (\underset{\sim}{r},0)$ at the initial time and a velocity $\underset{\sim}{v}_+ (t)$ for $t < 2t_o$. Consider a <u>second</u> system starting at time t_o with an initial velocity $\underset{\sim}{v}_- (t_o) \equiv -\underset{\sim}{v}_+ (t_o)$. By reversibility the evolution of this second system over the interval $(t_o, 2t_o)$ is related to the velocity of the first system over the interval $(0, t_o)$ by

$$\underset{\sim}{v}_-(t) = -\underset{\sim}{v}_+(2t_o - t) \quad ; \quad t_o < t < 2t_o \tag{14}$$

Calling $v^n = \underset{\sim}{vv} - - -$, we have

$$v^n_-(t) = (-1)^n \, v^n_+(2t_o - t) \quad ; \quad t_o < t < 2t_o \tag{15}$$

Now consider an ensemble of systems, each member having a different $\underset{\sim}{v}_+(0)$. As in the three-point method we consider cases for which $\langle \underset{\sim}{v}_+(t) \rangle = 0$. At t_o construct a second ensemble formed by taking for each member of the ensemble $\underset{\sim}{v}_-(t_o) = -\underset{\sim}{v}_+(t_o)$. It follows from (15) that

$$\langle v^n_-(t) \rangle = (-1)^n \langle \underset{\sim}{v}^n_+(2t_o - t) \rangle \tag{16}$$

$$t_o < t < 2t_o$$

Special cases of (16) for n=2 and 3 are

$$\langle U_-(t) \rangle = \langle U_+(2t_o - t) \rangle \quad ; \quad \langle T_-(t) \rangle = - \langle T_+(2t_o - t) \rangle \tag{17}$$

We compare the evolution of the two ensembles starting from t_o over the interval $(t_o, 2t_o)$. At the initial time $t = t_o$, from (17), both ensembles have the same energy $\langle U_-(t_o) \rangle = \langle U_+(t_o) \rangle$ and the ternary correlation functions are the negative of each other $\langle T_-(t_o) \rangle = -\langle T_+(t_o) \rangle$. From (16) with $t = t_o$ all even moments are equal and all odd moments are the negative of each other. Assuming that the ensemble at t_o does not weight $\underset{\sim}{v}$ and $-\underset{\sim}{v}$ equally (e.g. Gaussian) these two ensembles, having different initial moments, evolve in different ways. The evolution of the second ensemble over $(t_o, 2t_o)$ is related to the evolution of the first ensemble over $(0, t_o)$ by (16). If the first ensemble at t_o were e.g. Gaussian then velocity reversible does not produce a new ensemble i.e., the first and second are the same ensemble. From the moment point of view all odd moments are zero and therefore all moments of the two ensembles are equal. Assuming that this is not the case the evolution of the two ensembles, even though they start with the same $\langle U \rangle$, can evolve in distinctly different ways. Suppose that the first ensemble evolves over

$(0, 2t_o)$ in such a way that the energy flows from large to small scales and therefore from large to small scales over $(t_o, 2t_o)$. The second ensemble energy over the internal $(t_o, 2t_o)$, by (17), flows from small to large scales. The usual inviscid quasi-normal theory[3] explicitly shows this type of behavior. The above argument is general in that it does not involve a specific theory.

For the three-point theory, $\langle W_+^{(3)}(t,t_o)\rangle$ and $\langle W_-^{(3)}(t,t_o)\rangle$ go to zero for $t = t_o + \tau_c$. According to (a) and (c) the right-hand side of the energy equation (2) for $t > t_o + \tau_c$ depends only on T_c and according to (b) T_c is a functional of $\langle U\rangle$. For the $\langle U\rangle$ of the two ensembles to evolve from t_o in the same way $T_{c+}(t)$ must essentially equal $T_{c-}(t)$. For the first ensemble T_{c+} is given by

$$T_{c+}(t) = -\int_{t_o}^{t} dt''\langle W_+^{(3)}(t,t'')\rangle\langle L_+^3(t'')T_+(t'')\rangle \quad ; \quad t_o < t < 2t_o \tag{18}$$

The contribution to the integral comes from an interval $(t-\tau_c) < t'' < t$ i.e., it depends on $\langle U_+(t'')\rangle$ for t'' in the neighborhood of $t'' = t$. For the second ensemble $T_{c-}(t)$ is

$$T_{c-}(t) = -\int_{t_o}^{t} d\tau\langle W_-^{(3)}(t,\tau)\rangle\langle L_-^3(\tau)T_-(\tau)\rangle \quad ; \quad t_o < t < 2t_o \tag{19}$$

By reversibility and the fact that L^3 is linear in $\underset{\sim}{v}$ we have

$$\langle L_-^3(\tau)T_-(\tau)\rangle = \langle L_+^3(2t_o-\tau)T_+(2t_o-\tau)\rangle$$

so that

$$T_{c-}(t) = -\int_{2t_o-t}^{t_o} dt''\langle W_-^{(3)}(t,2t_o-t'')\rangle\langle L_+^3(t'')T_+(t'')\rangle \tag{20}$$

The contribution to the integral comes from an interval $(2t_o-t) < t'' <$

$(2t_o-t) + \tau_c$ i.e., it depends on $\langle U_+(t")\rangle$ for t" in the neighborhood $t"=2t_o-t$. For a system (the first ensemble) that is evolving over an interval $(0, 2t_o)$ the value of $\langle U_+\rangle$ for a time in the interval $(0, t_o)$ is different from its value for a time in the interval $(t_o, 2t_o)$ and therefore T_{c+} is in general different than T_{c-}.

Our conclusion is that for a given initial $\langle U\rangle$ there do exist ensembles (related by velocity reversible) of differing initial $\langle T\rangle$ that evolve in distinctly different ways over a time interval t_o. The above argument restricts the initial $\langle T\rangle$ for which the three-point method is valid. There can be a class of ensembles with the same initial $\langle U\rangle$ but different initial $\langle T\rangle$ that evolve in essentially the same way over the interval t_o. However the class cannot contain pairs of ensembles that are related by velocity reversal.

Acknowledgment

This work was supported by the United States Office of Naval Research.

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APPENDIX B

Comment on

"Adiabatic Modifications to Plasma Turbulence Theories"

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In Ref. 1 Catto introduced a new perturbation scheme to account for the adiabatic response of a plasma and obtained the nonlinear dielectric function. Krommes[2] calculated the dielectric function from the conventional theory obtaining a result differing from Catto's by a term $(k^2D\tau^2)$ and then remarked that "Catto makes subsidary approximations which lead him to neglect the $(k^2D\tau^2)$ term". Catto[3] responded that the term $(k^2D\tau^2)$ appears erroneously in Ref. 2 "because the properties of the adiabatic response are not properly preserved by the average trajectories". The purpose of this note is to show that Catto's new scheme can be obtained from the conventional scheme if use is made of the constraint

$$LF_M = 0 \tag{1}$$

where F_M is the part of the distribution function that depends on the energy, $L=\bar{L}+L'$, $\bar{L}\equiv \underset{\sim}{v}\cdot \frac{\partial}{\partial \underset{\sim}{r}}$, $L'\equiv \frac{e}{m}\underset{\sim}{E}'\cdot \frac{\partial}{\partial \underset{\sim}{v}}$ and $\underset{\sim}{E}'=\nabla\phi$ is the fluctuation in the field. The ensemble average field $\bar{E}$ is zero.

We start with Weinstock's[4] formulation of the conventional scheme for f, the fluctuating part of the distribution function F

$$f(\underset{\sim}{r},\underset{\sim}{v},t) = U_A(t,0)f(0) - \int_0^t dt' U_A(t,t')L'(t')\bar{F}(t') \tag{2}$$

where $F=\bar{F}+f$. The operator U_A is defined by

$$[\frac{\partial}{\partial t} + P]U_A(t,t_o) = 0 \tag{3}$$

where $P\equiv-(A-1)L$ and A is an operator which takes the ensemble average of everything on which it operates. Equation (2) is exact and therefore can be employed in the problem of the near equilibrium plasma. In this case[1]

$F=F_M+G$, $F_M=F_o\exp[\frac{e\phi}{T}]$, $F_o=c\exp[-mv^2/2T]$. We also have $f=F'_M+g$ where $F'_M\equiv(F_M-\bar{F}_M)$ and $g=(G-\bar{G})$. Equation (2) is then

$$f = U_A(t,0)[F'_M(0)+g(0)] - \int_0^t dt'U_A(t,t')L'\bar{G} - I \tag{4}$$

$$I \equiv \int_0^t dt'U_A(t,t')L'\bar{F}_M(t')$$

The approximation developed by Weinstock[4] for U_A is

$$U_A(t,t')(1-A) \approx (1-A)\bar{U}(t,t') \tag{5}$$

where $\bar{U}$ is the ensemble average of the Vlasov operator U. The problem referred to by Catto[1] is in using $\bar{U}$ in I since F_M satisfies (1). This problem can be circumvented by a transformation on I. The integral I can be written as

$$I = \int_0^t dt'U_A(t,0)U_A^{-1}(t',0)L'(t')\bar{F}_M(t')$$

where U_A^{-1} is the inverse of U_A and satisfies the equation

$$\frac{\partial}{\partial t}U_A^{-1}(t,t_o) - U_A^{-1}(t,t_o)P = 0 \tag{6}$$

From $\overline{L\,F_M}=0$ it follows that $\bar{L}\,\bar{F}_M=-\overline{L'F'_M}$ and from this that $L'\bar{F}_M=-PF'_M$. Using this relation and (6) in I we have

$$I = U_A(t,0)F'_M(0) - F'_M(t) + \int_0^t dt'U_A(t,t')\frac{\partial F'_M(t')}{\partial t'} \tag{7}$$

Equation (4) is then, using (5)

$$f = \bar{U}(t,0)g(0) + F'_M(t) - \int_0^t dt'\bar{U}(t,t')[\frac{\partial F'_M(t')}{\partial t'} + L'\bar{G}] \tag{8}$$

Equation (8) is equivalent to Catto's[1] eq. (6b) except that the operator $\bar{U}$ acts on the entire term $\frac{\partial F'_M}{\partial t'}$ and not just the $\frac{\partial \phi}{\partial t'}$ part. Using the same approximation as Catto, following his eq. (6b) we have[3]

$$\tilde{f}_k \equiv \int d\underset{\sim}{v}f_k \approx \frac{e}{T}\phi_k(t)[1+i\omega\int d\underset{\sim}{v}d\underset{\sim}{v}_o U_{k,\omega}(\underset{\sim}{v},\underset{\sim}{v}_o)F_o(\underset{\sim}{v}_o)] \approx$$

$$\approx \frac{e}{T}\phi_k(t)\int d\underset{\sim}{v}_o F_o(\underset{\sim}{v}_o)\{1+\int_0^\infty d\tau(i\omega)\exp[i(\omega-kv_o\tau)-k^2D\tau^3/3]\} \tag{9}$$

where $U_{k,\omega}$ is defined in ref. (2). The conventional result is obtained from (2) using (5) and replacing $\bar{F}$ by F_o and results[2] in (9) with the factor

($i\omega$) replaced by ($i\omega - k^2 D_T^2$). The conventional perturbation (2) in conjunction with the constraint (1) gives the result of Catto (8) and produces in the explicit form for $\tilde{f}$ a term differing by ($k^2 D_T^2$) from the explicit form for $\tilde{f}$ obtained only from the conventional perturbation (2).

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